Short time large deviations of the KPZ equation

Yier Lin

Columbia University

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Joint work with Li-Cheng Tsai (Rutgers)

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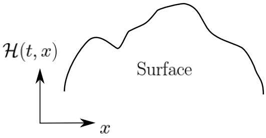
KPZ equation

- Introduced by Kardar-Parisi-Zhang (1986)

$$\partial_t \mathcal{H}(t,x) = \frac{1}{2}\partial_{xx}\mathcal{H}(t,x) + \frac{1}{2}\big(\partial_x \mathcal{H}(t,x)\big)^2 + \xi(t,x)$$



- Stochastic Heat Equation

$$\partial_t \mathcal{Z}(t,x) = \frac{1}{2}\partial_{xx}\mathcal{Z}(t,x) + \xi(t,x)\mathcal{Z}(t,x)$$

- Take $\mathcal{Z} = e^{\mathcal{H}}$, formally $\mathcal{Z}$ solves the Stochastic Heat Equation.
- Hopf-Cole solution $\mathcal{H}(t,x) := \log \mathcal{Z}(t,x)$.
- Narrow wedge initial data: $\mathcal{Z}(0,x) = \delta_0(x)$

Short time behaviors of KPZ

- [Amir-Corwin-Quastel 11]

$$\frac{\mathcal{H}(2t,0)+\frac{t}{12}}{t^{\frac{1}{3}}} \Longrightarrow \text{Tracy Widom GUE} \quad t\to\infty$$

$$\frac{\mathcal{H}(2t,0)+\log(\sqrt{4\pi t})}{t^{\frac{1}{4}}} \Longrightarrow \mathcal{N}(0,\sqrt{\frac{\pi}{2}}), \quad t\to 0.$$

  - What about the Large Deviation Principle (LDP)?

- Want to understand as $t\to 0$, how $\mathbb{P}(\mathcal{H}(2t,0)<-s)$ decays in s.
  - [Kolokolov-Korshunov 07] predicts $\frac{5}{2}$-power law in deep lower tail
  - Proved for large t: [Corwin-Ghosal 18] [Tsai 18] [Cafasso-Claeys 19]

- What we do:
  - Establish a LDP for the KPZ equation as $t\to 0$
  - Extract the $\frac{5}{2}$-power from the lower tail rate function

Main Result

Theorem (L., Tsai 20)

There exists $\phi+, \phi_-$ such that for $s > 0$,

$$\lim_{t\to 0} \sqrt{t} \log \mathbb{P}\big(\mathcal{H}(2t,0) + \log(\sqrt{4\pi t}) > s\big) = -\phi_+(s)$$

$$\lim_{t\to 0} \sqrt{t} \log \mathbb{P}\big(\mathcal{H}(2t,0) + \log(\sqrt{4\pi t}) < -s\big) = -\phi_-(s)$$

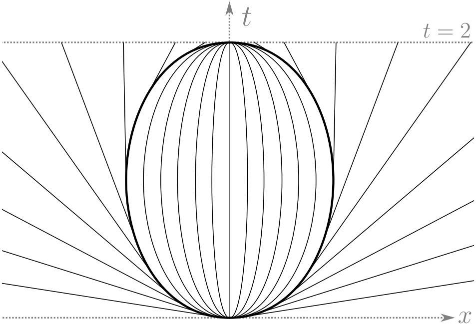
In addition,

$$\lim_{s\to 0} \frac{\phi_\pm(s)}{s^2} = \frac{1}{\sqrt{2\pi}}, \qquad \lim_{s\to\infty} \frac{\phi_-(s)}{s^{\frac{5}{2}}} = \frac{4}{15\pi}$$

- Do not rely on the exact formula.
- Method also works for the flat initial data.

A phenomenon

- Classify the geodesics of a directed polymer, non-uniqueness



- Phenomenon reminiscent of [Basu-Ganguly-Sly 17]